Analysis of adjacency effects for two Lambertian half-spaces

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Introduction

Abstract:
An analysis of one dimensional adjacency effects is presented. This classical problem is shown to be amenable to closed form error analysis when 1D adjacency correction schemes are used. In particular the error made when surface reflectance is retrieved using an infinite target assumption is given in closed form. This allows deductions for the behavior of the error as a function of wavelength and optical thickness. Typical length scales of the adjacency effects are deduced and the range and magnitude of the error are also given in closed form.

Background:
The problem of atmospheric correction for remote sensing data over heterogeneous terrain has been challenging since the advent of high resolution mappers. Numerous theoretical and numerical studies have been devoted to the topic with an emphasis on simulating the effect of terrain heterogeneity on the top of the atmosphere radiance. A useful approximation of adjacency effects was proposed in the eighties by Tanre where Monte-Carlo simulations were used to account for the contribution of pixels neighboring the line of sight pixel. This formulation of the adjacency effect is an approximation to the rigorous adjacency problem. However, it has the advantage of being easily implementable and has been used to correct ASTER and LANDSAT data for adjacency effects. It is based on the calculation of an environment reflectance, which captures the influence of the atmosphere and neighboring pixels on the top of the atmosphere signal. This environment reflectance is computed as the average of the surface reflectance around the line of sight pixel weighted by the point spread function of the atmosphere. The point spread function derived from Monte-Carlo simulations consists the molecular scattering or Rayleigh part and the aerosol part. The interesting feature of the point spread function derived in that way is that it is a sum of exponentials as a function of distance to the line of sight pixel. This feature of the point spread function is the key to our error analysis of the classical one dimensional or coastal adjacency problem. For such functions, the environment reflectance and the error in the infinite target approximation can be evaluated in closed form. From this the range and magnitude of the adjacency effects as a function of wavelength and aerosol optical thickness can easily be estimated.

Lambertian vs. Adjacency Formulation

Lambertian Approximation:
The top of the atmosphere reflectance ρ_{toa} for an infinite homogeneous Lambertian surface is given by Chandrasekhar's formula:

$$\rho_{toa} = \rho_{atm} + \frac{T^{\downarrow}T^{\uparrow}\rho_{Lamb}}{1 - S\rho_{Lamb}}$$

$T^{\uparrow}$ upward transmittance, $T^{\downarrow}$ downward transmittance, ρ_{atm} atmospheric reflectance, S spherical albedo. Knowing the atmospheric path radiance and atmospheric transmittance, the Lambertian reflectance is easily retrieved using:

$$\gamma = \frac{\rho_{toa} - \rho_{atm}}{T^{\downarrow}} \quad \Longrightarrow \quad \boxed{\rho_{Lamb} = \frac{\gamma/T^{\uparrow}}{1 + S\gamma/T^{\uparrow}}}$$

The downward transmittance can be split in direct and diffuse components: $T^{\uparrow}(\mu,\phi) = e^{-\tau/\mu} + t^{\uparrow}(\mu,\phi)$
So the lambertian approximation can expressed as:

$$\rho_{toa} = \rho_{atm} + \frac{T^{\downarrow}(e^{-\tau/\mu}\rho_{Lamb} + t^{\uparrow}\rho_{Lamb})}{1 - S\rho_{Lamb}}$$

Adjacency Approximation:
The difference between the Lambertian and the adjacency approximations stems from the treatment diffuse scattering term. The adjacency approximation accounts for scattering by neighboring pixels by multiplying the diffuse transmittance by a local average reflectance $<\rho>$

$$\rho_{toa} = \rho_{atm} + \frac{T^{\downarrow}(e^{-\tau/\mu}\rho + t^{\uparrow} <\rho>)}{1 - S <\rho>}$$

In this case the target reflectance ρ is given by $\boxed{\rho = \frac{\gamma(1 - S <\rho>) - t^{\uparrow} <\rho>}{e^{-\tau/\mu}}}$

Error between Lambertian and Adjacency retrieved reflectances

The reflectance retrieved in the Lambertian approximation ρ_{Lamb} will differ from the reflectance retrieved from the adjacency formulation ρ . The difference is a simple function of $<\rho> - \rho$

$$\boxed{\rho_{Lamb} - \rho = \frac{(t^{\uparrow} + Se^{-\tau/\mu}\rho)(<\rho> - \rho)}{T^{\uparrow} - Se^{-\tau/\mu}(<\rho> - \rho)}}$$

Close form expression of $<\rho> - \rho$
The average reflectance is defined by

$$<\rho>(x_o, y_o) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \rho(x_o - x, y_o - y)m(x,y)dxdy$$

$$m(x,y) = \frac{f(\sqrt{x^2+y^2})}{2\pi\sqrt{x^2+y^2}} \qquad f(r) = \sum_{i=1}^{4} a_i \exp(-\alpha_i r) \qquad \boxed{f(r) = F'(r)}$$

$$\begin{cases} F(r) = \frac{t_d^R F_R(r) + t_d^A F_A(r)}{t_d^R + t_d^A} \quad \text{with} \quad t_d^R = \frac{\tau_R}{2} \quad t_d^A = \frac{5\tau_A}{6} \\ F_R(r) = 1 - 0.930\exp(-0.08r) - 0.070\exp(-1.1r) \\ F_A(r) = 1 - 0.375\exp(-0.2r) - 0.625\exp(-1.83r) \end{cases}$$

Tanré *et. al*

$$\begin{cases} F(r) = \frac{t_d^R F_R(r) + t_d^A F_A(r)}{t_d^R + t_d^A} \\ F_R(r) = 1 - 0.930\exp(-0.08r) - 0.070\exp(-1.1r) \\ F_A(r) = 1 - 0.448\exp(-0.27r) - 0.552\exp(-2.83r) \end{cases}$$

6S algorithm

$$\begin{cases} F(r) = t_d^R F_R(r) + t_d^A F_A(r) \\ F_R(r) = 1 - 0.916\exp(-0.0821r) - 0.084\exp(-0.51r) \\ F_A(r) = 1 - 0.326\exp(-0.234r) - 0.674\exp(-3.1r) \end{cases}$$

Reinersman and Carder

Study in one dimension

In one dimension, since f(r) is a **sum of exponentials** the integral is in fact a sum of modified Bessel functions K_o

$$<\rho>(x_o) = \frac{1}{\pi}\sum_{i=1}^{4} a_i \int_{-\infty}^{+\infty} \rho(x_o - x)K_o(\alpha_i|x|)dx$$

For a two Lambertian half-spaces: $\rho(x) = \begin{cases} \rho_- & x < 0 \\ \rho_+ & x > 0 \end{cases}$

The error can be explicitly written as: $\boxed{<\rho>(x_o) - \rho(x_o) = -sgn(x_o)\left(\frac{\rho_+ - \rho_-}{2}\right)\sum_{i=1}^{4}\frac{a_i}{\alpha_i}\left(1 - \frac{2}{\pi}\int_0^{\alpha_i|x_o|} K_o(x)dx\right)}$

The error depends on the magnitude of the discontinuity $\frac{\rho_+ - \rho_-}{2}$ and also on the function:

$$\Lambda(x_o) = \sum_{i=1}^{4}\frac{a_i}{\alpha_i}\left(1 - \frac{2}{\pi}\int_0^{\alpha_i|x_o|} K_o(x)dx\right)$$

which captures the atmospheric effects. This is reflected in the coefficients a_i and α_i which depend on the parameter β defined by

$$\boxed{\beta = \frac{t_d^A}{t_d^R + t_d^A}}$$

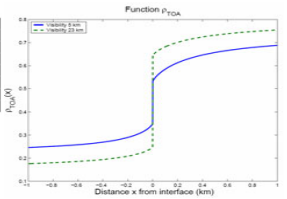
This comes from the definition of the influence function F(r)

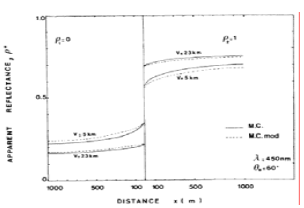
$$F(r) = \frac{t_d^R F_R(r) + t_d^A F_A(r)}{t_d^R + t_d^A} = (1-\beta)F_R(r) + \beta F_A(r)$$

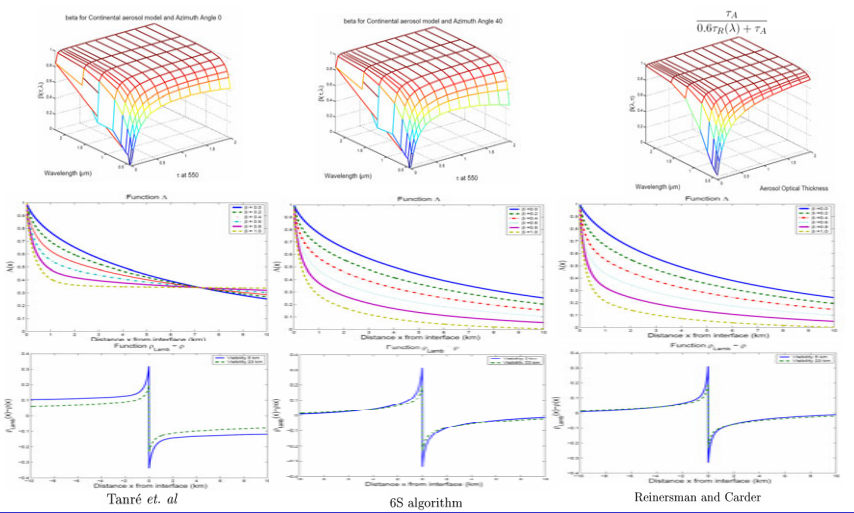
Numerical results

Atmospheric Parameters	Visibility 23 km	Visibility 5 km
τ_R ($\lambda = 0.45\ \mu$ m)	0.2157	0.2157
τ_A	0.2807	0.9306
ρ_{atm}	0.1281	0.2027
S	0.2128	0.3080
$t^{\uparrow}$	0.3707	0.5206
$T^{\downarrow}$	0.58	0.48
β	0.684	0.878

from *Tanre et al.* Applied Optics, 1979.







Tanré *et. al* | 6S algorithm | Reinersman and Carder

Conclusions

This study derived a closed form error analysis of the one-dimensional adjacency problem. The range of the adjacency effects as a function of wavelength and aerosol optical thickness can be estimated through a simple formula and the parameter β. The error made in neglecting adjacency effects and using an infinite target correction is given in closed form. Well-known contrast reduction effects found in the infinite target approximation are shown analytically and predictions for the range and magnitude of the error are given in closed form.